

# Points, Walls and Loops in Resonant Oscillatory Media

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## Abstract

In an experiment of oscillatory media, domains and walls are formed under the parametric resonance with a frequency double the natural one. In this bi-stable system, nonequilibrium transition from Ising wall to Bloch wall consistent with prediction is confirmed experimentally. The Bloch wall moves in the direction determined by its chirality with a constant speed. As a new type of moving structure in two-dimension, a traveling loop consisting of two walls and Neel points is observed.

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Oscillatory media spontaneously form in a wide variety of systems as they are driven away from equilibrium. Examples are seen in many different fields [1,2,3]. In nonlinear oscillators having a small number of degrees of freedom, one of the central problem is to clarify synchronization and resonance. However, inspite of the development of theory and experiment, relatively little is known about the response to the external periodic forcing for spatially coupled oscillators. The simplest but unresolved case is parametric resonance of the system where nontrivial domain walls are predicted to exist [4]. In the parametric resonance, when the external forcing has the frequency double the natural frequency, two locked states are possible. The domain wall appears between two different locked states. In this system, sustained motions of walls are predicted including translational motion of wall, and oscillating walls due to the non-variational effect [4,5] even if two domains are symmetric. This is the crucial difference from domain walls in equilibrium systems whose dynamics is simply a relaxational process governed by a free energy. We present an experimental investigation of parametrically forced oscillatory media. A structural transition of domain walls associated with chirality breaking is elucidated. Furthermore, traveling loops consisting of two types Bloch walls and Neel points are observed.

Oscillatory media are realized by a convective cellular structure called Oscillating Grid Pattern(OGP) [6] in liquid crystal convection [7]. We use a nematic liquid crystal, 4-methoxybezyliden-4'-butylaniline (MBBA) doped with 0.01wt% of ion impurity, tetra-n-butylammonium bromide, to control the electrical conductivity. This liquid crystal is filled in a cell ( $2\text{cm} \times 2\text{cm} \times 50\mu\text{m}$ ) sandwiched between transparent electrodes. The temperature of the cell is controlled to  $25 \pm 0.01^\circ\text{C}$ . Applying an alternative (AC) voltage of  $V \sim 60[\text{V}]$  with frequency  $\omega/2\pi \sim 900[\text{Hz}]$ , we obtain stationary Grid Pattern(GP). It gives rise a two-dimensional lattice of about  $400 \times 400$  rectangle convective cells. Nematic director is stationally since the relaxation time is much longer (about 0.2sec) than the period of external AC field. [7] It is visualized by a shadowgraph under the microscope with polarized light. The shadowgraphic image intensity is directly related to the orientation of molecules of nematics. In this pattern sinks and sources form the centered rectangle net as shown in

Fig. 1(a). Slight increase of voltage causes the oscillatory instability of the Grid Pattern with natural frequency  $\omega_0$ ;  $\omega_0/2\pi \sim 1.3$ [Hz] which is independent of the external frequency  $\omega$ . We call this pattern OGP, from the fact that the positions of sinks and sources move oscillatingly. This is a good candidate of two-dimensionally distributed oscillatory media.

To observe behavior of a parametric resonance of the oscillator lattice, we modulate the AC voltage with nearly double the natural frequency,  $V_m = 2\sqrt{2}V(1 + r \cos \omega_e t) \cos \omega t$ , where  $r$  is the modulation ratio and  $\omega_e = 2\omega_0 + \Delta\omega$  and the detuning  $\Delta\omega$  is small. Typical patterns observed in a phase locked state under parametric resonance are shown in Figs.1(b) and (c). The dark lines are interfaces between two phase locked states. Across the dark line the phase of the oscillation jumps by  $\pi$  as will be shown later. The interfaces are walls in dynamical systems. Unlike in equilibrium systems, these walls can exhibit transition from stationary to propagating ones by varying the control parameters;  $r$  and  $\omega_e$ . We show a phase diagram in Fig.2(a) with varying them. Here the AC voltage for convection was fixed at  $\omega/2\pi = 928$ [Hz],  $V = 66.3$ [V], which corresponds to a little above the onset of the oscillatory instability of GP ( $\mu \equiv (V - V_c)/V_c = 0.009 \pm 0.001$  where  $V_c$  is the critical voltage for Hopf bifurcation). In the right half part of phase locked region, the wall exhibits stationary spatial periodic patterns (stripe) as shown in Fig.1(c). Recently this periodic pattern of walls (stripe) are studied theoretically [9,10,11]. As we decrease the modulation frequency, the wavelength of the pattern increases, and finally results in isolated walls in the left half part of the phase locked region as shown in Fig.1(b). Here we focus on this region in which stationary or propagating isolated walls are observed. To elucidate the transition from stationary to propagating walls, we measure the velocity of moving wall as a function of modulation frequency  $\Delta\omega$  (Fig.2(b)) with fixing  $r = 0.13$  (Fig.2(b)). It indicates that the transition is a second order consistent with prediction [4].

It was predicted that the spontaneous breaking of chirality is responsible for the transition from stationary to moving walls in nonequilibrium systems [4]. The stationary one is called Ising walls and the moving one is Bloch walls, relying on the analogy to an anisotropic X-Y model [8]. If the observed stationary interfaces are Ising walls, the amplitude of oscillation

must vanish where the phase jumps. Furthermore if moving interfaces are Bloch walls, the amplitude does not vanish at the core, but two domains are connected by rotating the vector of the complex order parameter of the oscillation. Now chirality of the wall is defined by the direction of this rotation, *i.e.*, right-handed or left-handed [4]. The moving direction of the wall is determined by the chirality. Hence, one can judge wall types by its amplitude at the core and chirality.

In order to clarify the structural transition of walls we perform analysis as followings. The image intensity  $\mathcal{G}(x, y, t)$  was digitized with a resolution of  $640 \times 480$  pixels and 256 grey scale levels at frequency of 15Hz. Here we choose  $x$ -axis to be parallel to the direction of the alignment of nematics.

Under the parametric resonance, OGP has temporal frequency  $\omega_0$  (1.3Hz), and lattice wavenumber  $(k_x, k_y) = (2\pi/63, 2\pi/105)(\mu\text{m}^{-1})$ . Thus at the lowest order one can expand  $\mathcal{G}(x, y, t)$  as follows,

$$\mathcal{G}(x, y, t) = [1 + a \exp i(\omega_0 t + \psi)] \exp(ik_x x + ik_y y) + c.c. + h.o.t., \quad (1)$$

where  $a$  represents the amplitude and  $\psi$  the phase of oscillation mode, and *h.o.t.* denotes higher order terms. These amplitude and phase are slowly varying functions in space and time, compared with lattice wave number and natural frequency. In order to obtain only the slow variations in the Eq.(1), we use a broad band pass filter centered at  $\omega_0$  in frequency and  $(k_x, k_y)$  in wavenumber. Thus we obtain filtered signal;  $a \exp(i\psi)$ , which is the complex order parameter of our system.

Although the oscillation and interfaces are two-dimensional phenomena, to show temporal evolutions of interfaces evidently at first we take a one-dimensional section of oscillators intersecting a wall perpendicularly. Figure 3 shows the result about a stationary wall. In Figure 3(a) the solid line represents the spatial variation of the phase, and there exists a jump about  $\pi$  in its center. At the same point the amplitude (dotted line) falls almost to zero. A spatio-temporal plot of amplitude profile in Figure 3(b) shows evidently that the wall is stationary. Therefore we conclude that stationary walls are Ising walls. On the other

hand, in Fig. 4 we show the result about the moving wall. The amplitude at the walls are relatively small but does not vanish (dotted line in Fig. 4(a)). Phase gradually changes by  $\pi$  at the center of wall (solid line in Fig. 4(a)) which slope is less steeper than the case of Ising wall. Notice that the wall is moving leftward in spatio-temporal plot of phase profile (Fig. 4(b)). Consequently it is concluded that moving walls are Bloch wall. As in an easy-axis ferromagnets, there are two kinds of Bloch walls in this system. The phase decreases from left domain to right one in Fig. 4. The other type of Bloch walls exhibiting rightward motion are also observed, which connects the same domains with increasing the phase. Hence, the motion of Bloch wall is determined by its chirality.

Let us discuss two-dimensional structures and behavior. In two dimension, Bloch walls can form closed loops. The simplest loop is the one consisting of only a single type of Bloch wall which expands invading outer region or shrinks invading inner region depending on its chirality. (see Fig. 5(a),(b)) However interesting motion is expected if the loop consists of two different types of Bloch walls jointed at two points. (Fig. 5(c)) This point is called Neel point or Connecting Point(CP) [5,13]. Some interesting phenomena are expected because of the interplay between two types of Bloch walls jointed by CP. Among them we report a new type of moving structure in two-dimensional space; a translational motion of a loop. Figure 6(a) shows a snapshot of a moving loop exhibiting translational motion to the right. The loop travels persistently to one direction until it collides with other walls or loops. We analyzed the two- dimensionally distributed oscillators near the upper end of the loop. Figure 6(b) shows the spatial phase variation. It is seen that two different Bloch walls are connected at a kind of branch cut of a  $2\pi$  jump. In Fig. 6, the left domain and the right domain are the same since they are connected out side the loop. The branch cut is resulted from this fact. Notice that the center domain is connected with right one by increasing the phase and with left one by decreasing. These increasing and decreasing correspond to left- and right-handed rotation of the vector of complex order parameter. Their directions of motions are opposite to each other, *i.e.* one invades outward of a loop and the other does inward. Hence, again the motion of Bloch wall is determined uniquely by its chirality.

Considering the topology of the loop, two CPs must exist at the upper and lower ends of the loop which correspond to the singular points of the branch cut. The amplitude of Bloch wall does not vanish, nevertheless at only CP it must vanish due to topological constraint. We plot the cross section of the amplitude profile by crossing interfaces twice at a Bloch wall and CP in Fig. 6(c). Evidently Fig. 6(c) shows that the amplitude is relatively small at Bloch wall, and zero at CP. This confirms that the traveling loop consists of two different types of Bloch walls and two CPs as schematically illustrated in Fig.5(c).

In the traveling loops, CPs move with the Bloch walls. This is in contrast with usual spiral patterns in which the core (CP) remains unmoved. [12,13,14] A possible reason is following. In the present experiment, the system is fully non- variational because the system consisting of limit cycle oscillators. In fact, the traveling loop is observed numerically in CGL equation with parametric forcing term by appropriate choice of the linear dispersion coefficient and the detuning parameter [15]. Recently observed traveling spots in a model of reaction diffusion equation [16] seems very similar to the present traveling loops. Therefore we believe that the phenomena are generic in (resonant) oscillatory media. The mechanism of the motion of CP may be related to core meandering in spiral patterns. It is an open problem in resonant system. The motion of the loops and connecting point will open an interesting question about dynamics of interfaces; walls, lines and points, appearing in higher dimensional space for nonequilibrium systems [17].

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## FIGURES

FIG. 1. The shadowgraphic image of the Grid Pattern. The length of bars corresponds to  $100\mu m$ . (a)A snapshot of oscillating Grid Pattern(OGP). (b)OGP under the parametric resonance. Dark lines are domain wall between two different locked states. (c)same as (b) but showing stripe pattern.

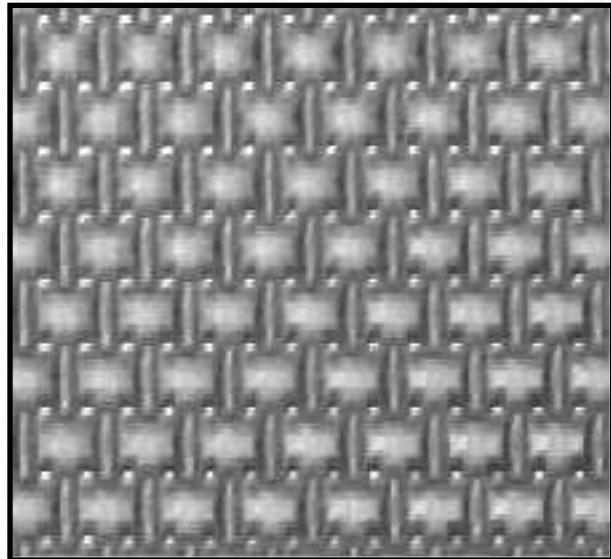
FIG. 2. (a) The phase diagram of the parametric resonance as a function of modulating ratio  $r$  and frequency  $\omega_e$ . (b) Traveling velocity of Ising and Bloch walls as a function of frequency detuning  $\Delta\omega$  with fixing  $r = 0.13$ .

FIG. 3. Stationary interface for one-dimensional oscillators along a line intersecting the wall. (a)the spatial variation of the amplitude (dotted line) and the phase (solid line) (b)space time evolution of the the amplitude profile. Black (white) pixels correspond to a small (large) amplitude of oscillation.

FIG. 4. Moving interface for one-dimensional oscillators along a line intersecting the wall. (a)the spatial variation of the amplitude (dotted line) and the phase (solid line) (b)space time evolution of the phase variation. The grey sale corresponds to white for 0 and black for  $2\pi$ .

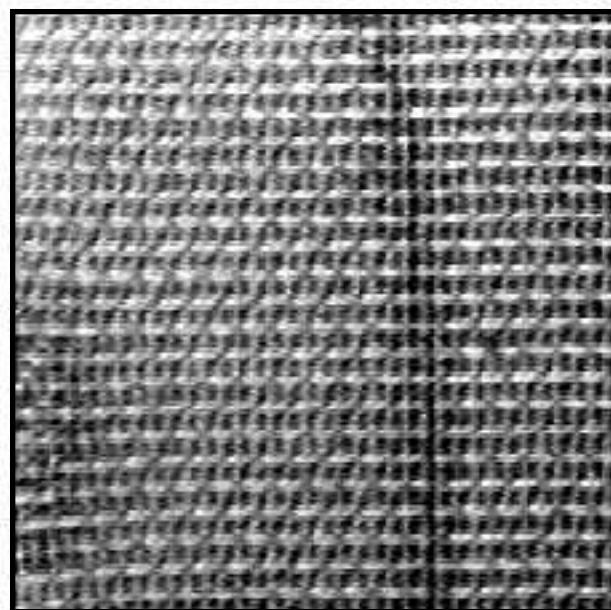
FIG. 5. Schematic illustration of possible types of loops. (a) An expanding loop consists of a Bloch wall (+). (b) A shrinking loop consists of a Bloch wall (-). (c) A traveling loop consists of two Bloch walls (+ and -) and Neel points(CPs).

FIG. 6. Experimental data of a traveling loop (a) Phase distribution of a snapshot of a loop exhibiting translational motion to rightward. The bar corresponds to  $100\mu m$ . (b) a spatial variation of the phase around upper end of the loop. (c) a cross section of amplitude profile which crosses a Bloch wall (B) and a CP. The amplitude vanishes at the CP.



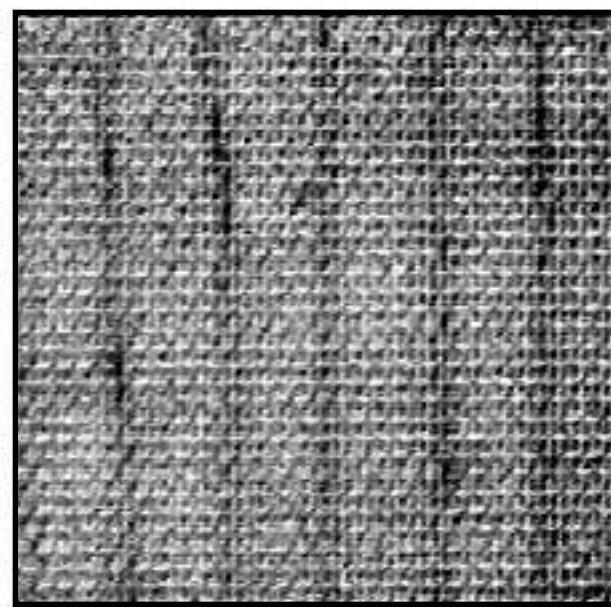
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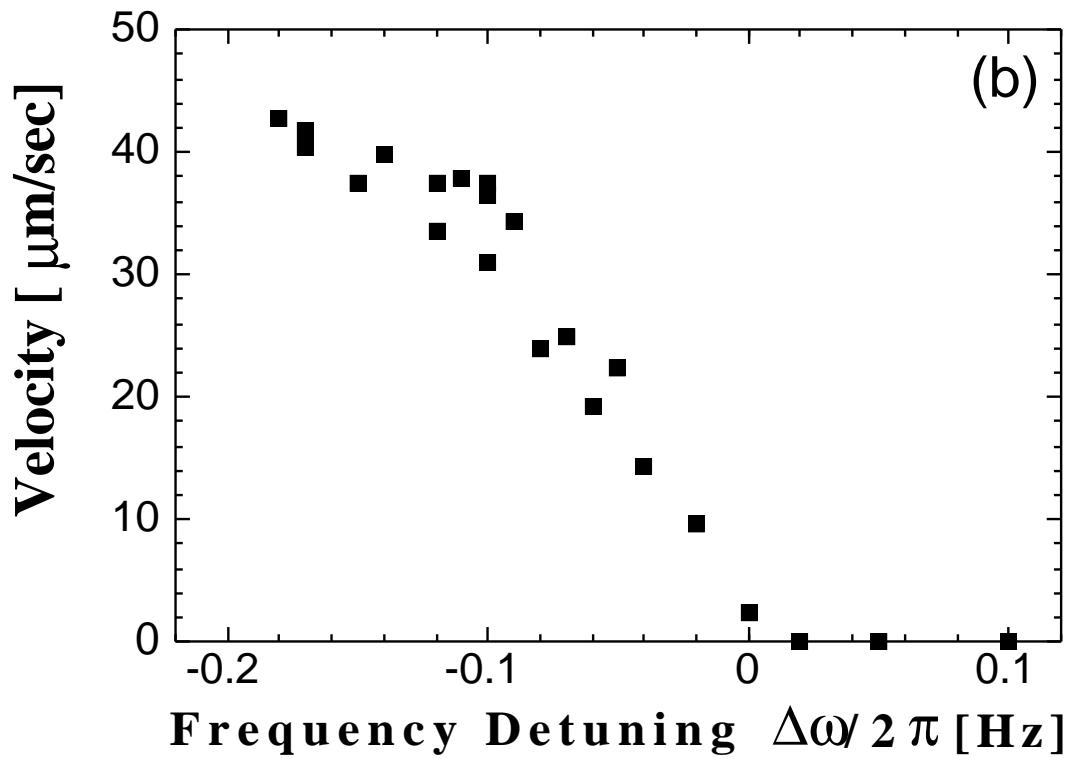
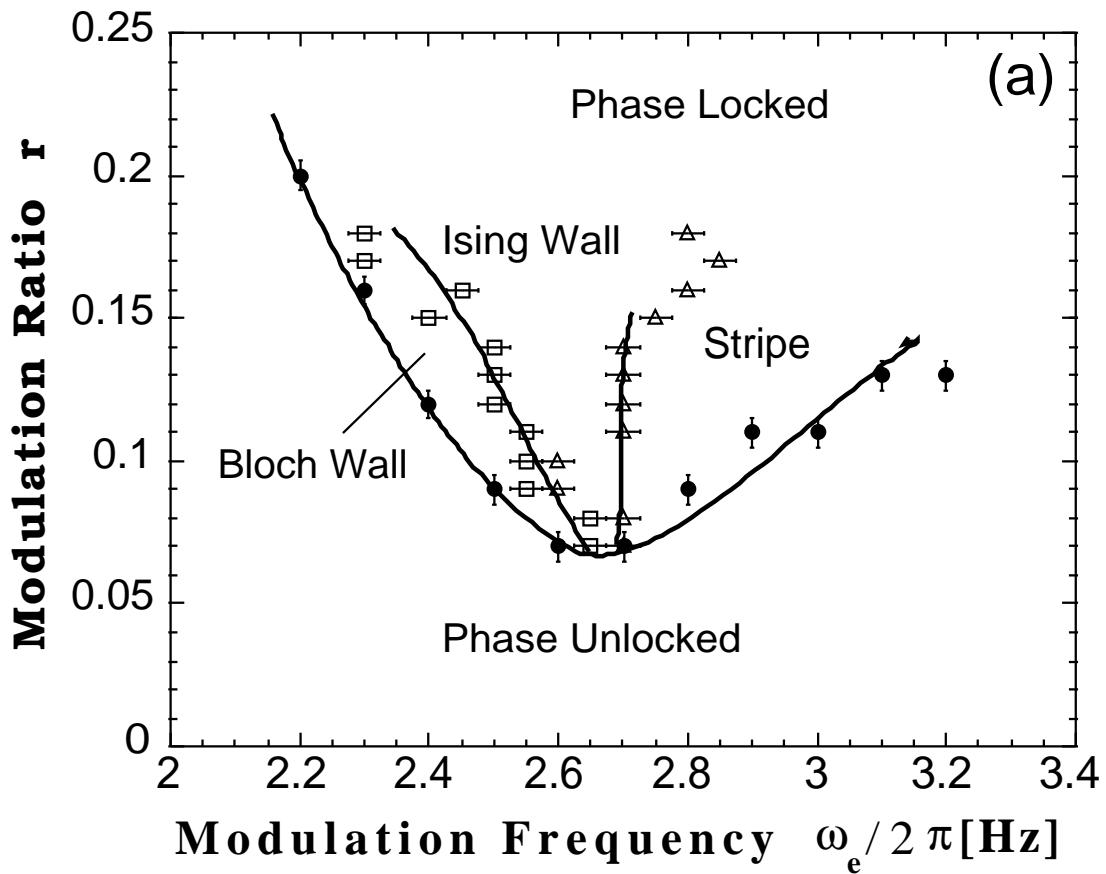
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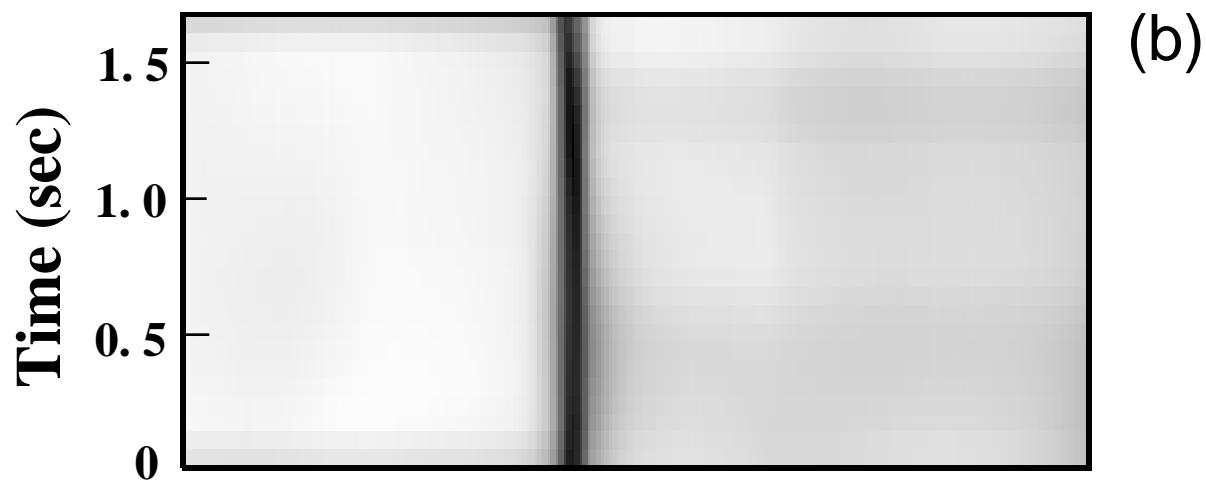
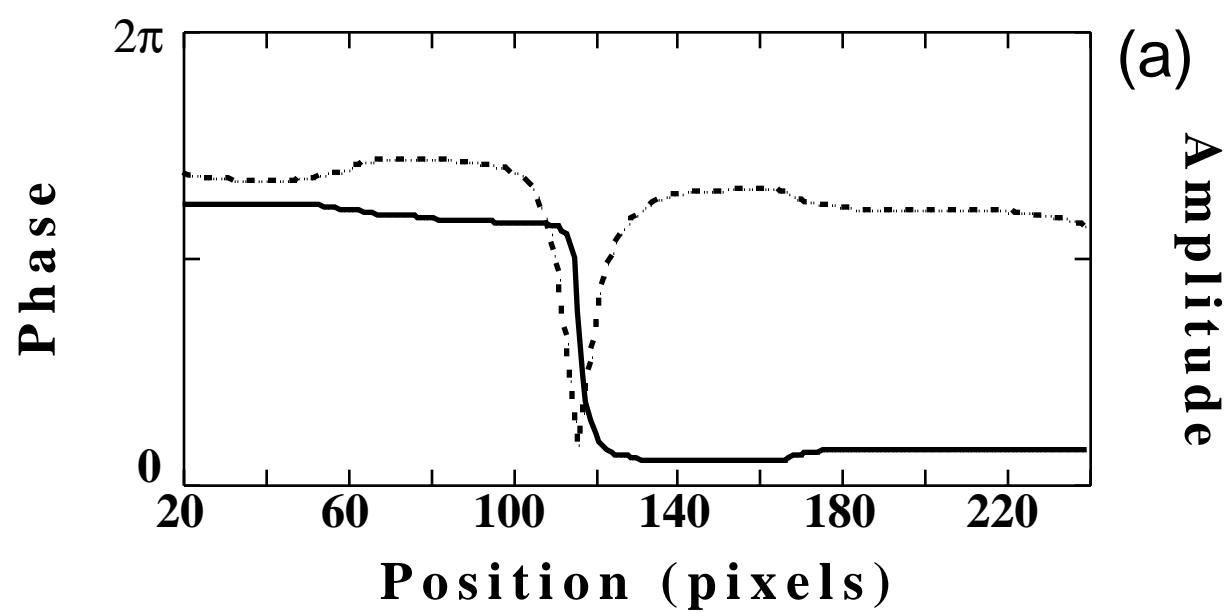
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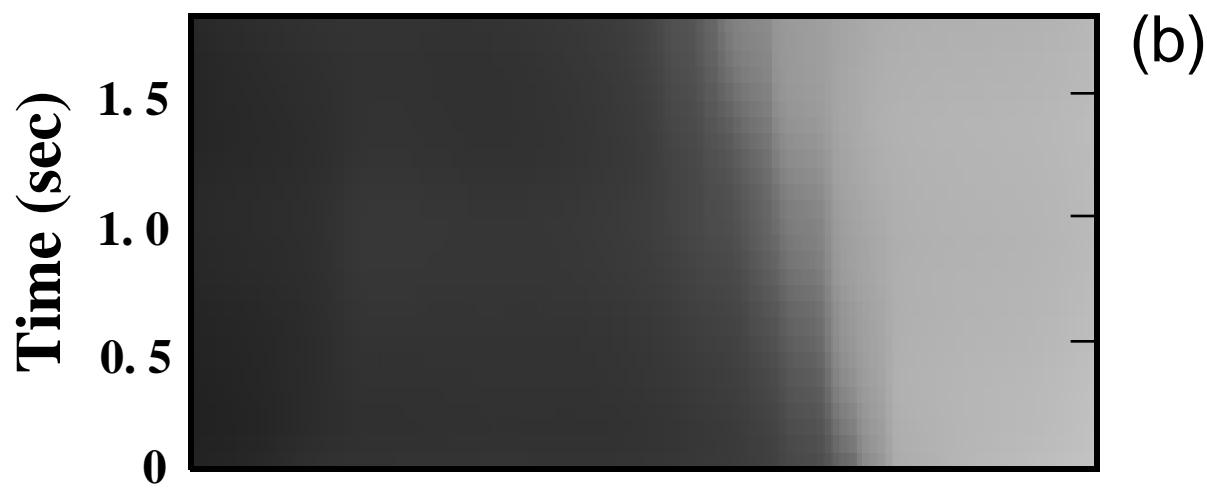
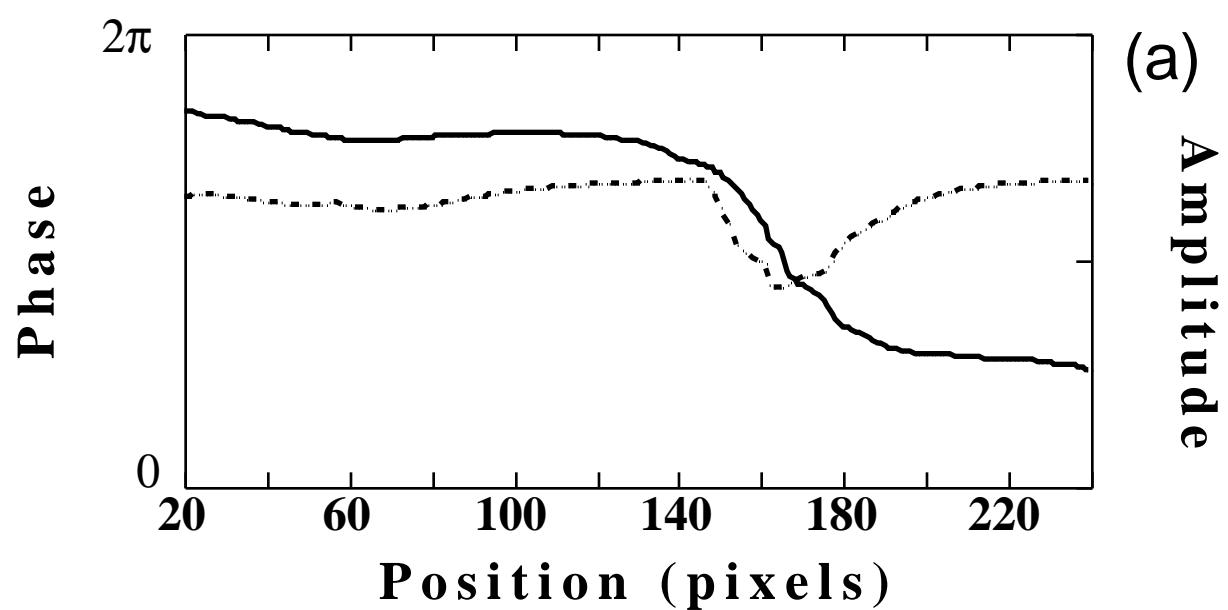


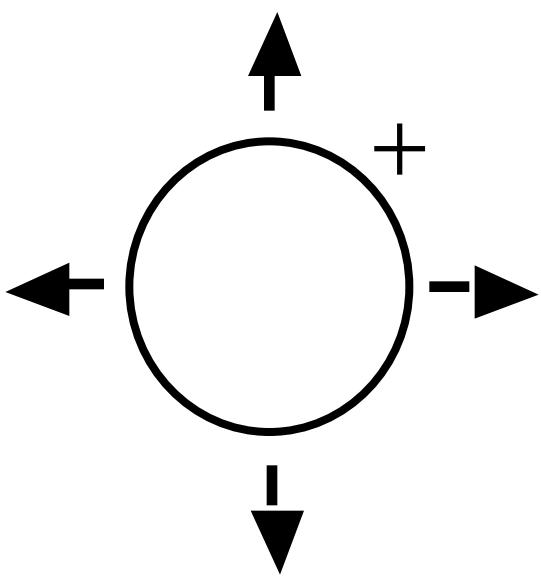
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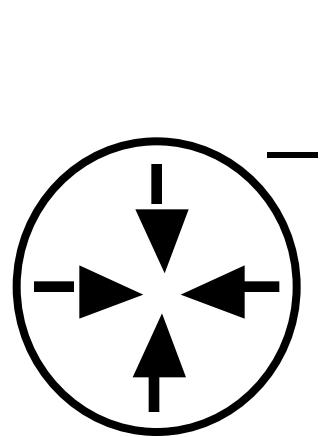




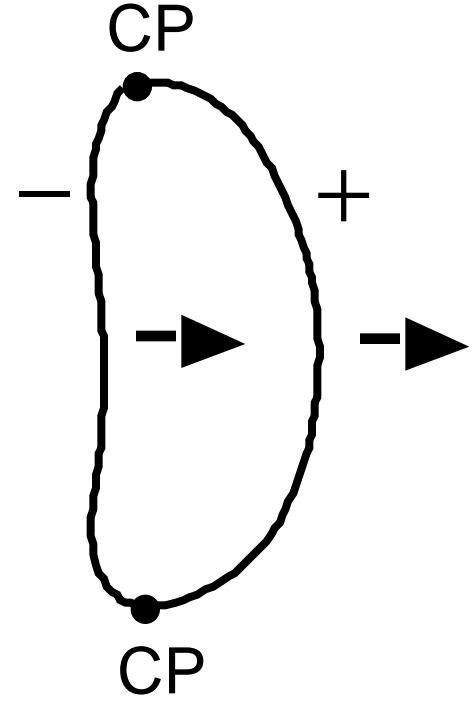




(a)



(b)



(c)

